Using Envelope and Matrix Codes (TRACE 3-D & TRANSPORT Examples-2)

George H. Gillespie

G. H. Gillespie Associates, Inc.
P. O. Box 2961
Del Mar, California 92014, U.S.A.

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Presentation Outline

4. Using TRACE 3-D & TRANSPORT to Solve Problems

⇒ **You will use the Simulation Lab computers in the classroom**

- **FODO Lattice**
- **Finding Matched Beam for a FODO Lattice**
- **Finding a FODO Lattice for a Matched Beam Requirement**
- **Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel ⇐ Fitting Requirements**
- **Transfer Line Matching (Fitting)**
- **Compare TRACE 3-D & TRANSPORT Transfer Line Matching (Fitting)**
- **A Few Other Representative Problems (Homework)**

Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

Using R-matrix Constraints to Solve Problems

- **Consider particle starting (point a) on-axis:** $x_a = 0$ **and** $y_a = 0$
- **Let the Reference Velocity** $= v_s$
- **The change in the x-coordinate at the end (point b) of a system** $[x_b]$ **due small initial velocities** $[v_{xa}, v_{ya}]$ **away from the axis is:**

$$x_b = R_{12}\,[x'_a] + R_{14}\,[y'_a]$$

where $x'_a = v_{xa} / v_s \approx (Px)_a / P_s$ **and** $y'_a = v_{ya} / v_s \approx (Py)_a / P_s$

- **Suppose we want a lens system that will bring a group of such on axis particles back to the x axis.**
  - $\Rightarrow$ **This could be accomplished for *ALL*** v_{xa} **and** v_{ya} **if (and only if):**

$$R_{12} = R_{14} = 0$$

- **The condition** $R_{12} = 0$ $\Rightarrow$ **"Point-to-Point" Focus in x**
- **Similarly if** $R_{34} = 0$ $\Rightarrow$ **"Point-to-Point" Focus in y**

Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

$\Rightarrow$ **Individual R-matrix Element Constraints Achieve Different Goals**

Some Useful R-Matrix Fitting Constraints

- **Trace of** R-Matrix **for stability in a periodic system:** $(1/2)\,|\mathrm{Tr}[R]| \leq 1$
- **For point-to-point optics in the horizontal (**x**) direction:** $R_{12} = 0$
- **For parallel-to-parallel optics in the horizontal (**x**) direction:** $R_{21} = 0$
- **For parallel-to-point optics in the horizontal (**x**) direction:** $R_{11} = 0$
- **For point-to-parallel optics in the horizontal (**x**) direction:** $R_{22} = 0$
- **For an achromatic system in the horizontal (**x**) direction:** $R_{16} = 0$ and $R_{26} = 0$
- **Similar conditions for the vertical (**y**) direction involving the** R_{yy} **submatrix**

Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

Other R-matrix Properties Useful for Constraint Conditions

- R_{11} **describes dependence of the output "size"** x_b **on input "size"** x_a**:**

  $\Rightarrow$ $R_{11} = M_x$ = x-Magnification ($|R_{11}|>1$) or Demagnification ($|R_{11}|<1$)

  Similarly:

  $\Rightarrow$ $R_{33} = M_y$ = y-Magnification ($|R_{33}|>1$) or Demagnification ($|R_{33}|<1$)

- R_{21} **describes dependence of the output angle** x'_b **on input "size"** x_a **:**

  $$R_{21} = -1 / f_x \quad \text{where } f_x = \text{x-Focal Length}$$

  Similarly:

  $$R_{43} = -1 / f_y \quad \text{where } f_y = \text{y-Focal Length}$$

- **If** $R_{21} < 0$ **then focusing in x, while** $R_{21} > 0$ **is defocusing in x**
- **If** $R_{43} < 0$ **then focusing in y, while** $R_{43} > 0$ **is defocusing in y**

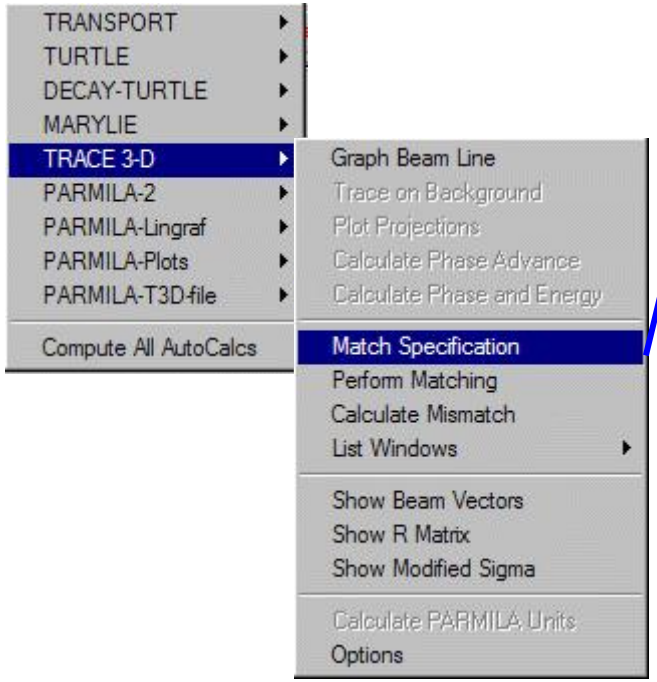
- **For a Quadrupole x and y not the same (**$R_{21} \neq R_{43}$**)** $\Rightarrow$ **astigmatic lens**

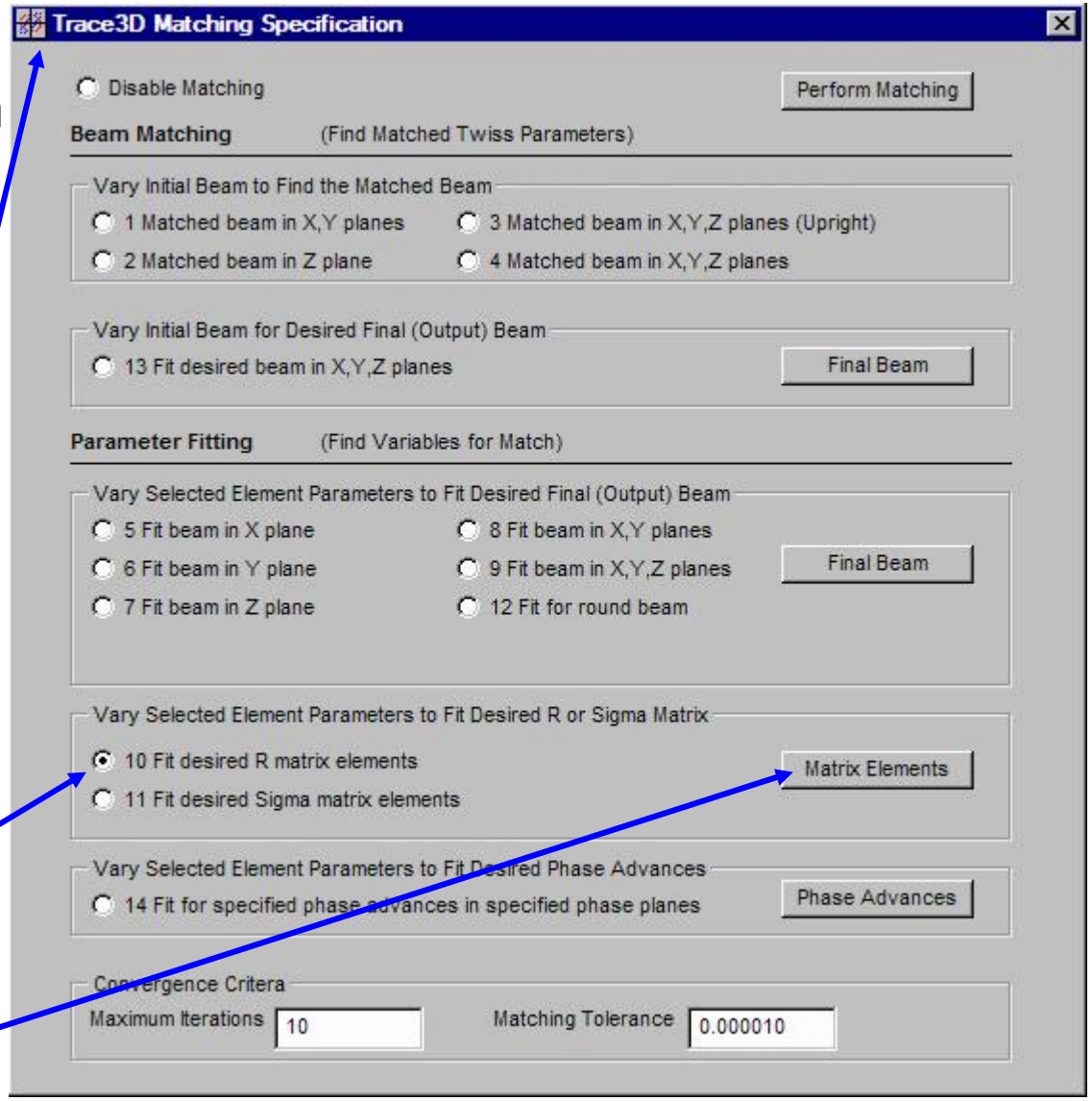
Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

4. Open the PBO Lab (homework) file saved as:

"Setup_3_FODO_1"

Open Match Specification





Select the R-Matrix Fitting Specification

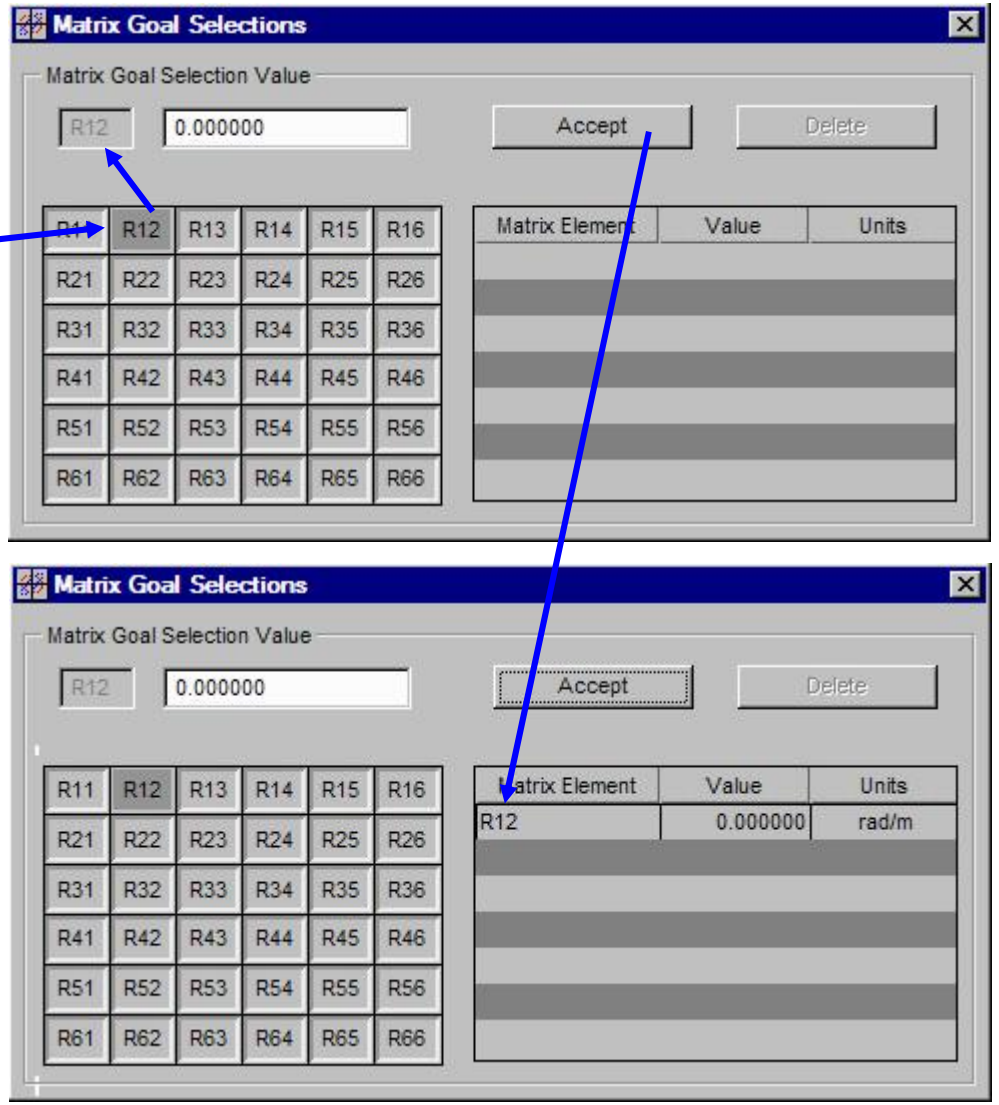
Open Matrix Elements

Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

Specify Point-to-Point Optics for x Direction:

a. Select the R12 Element

b. Hit "Accept" Button



Bonus Credit!

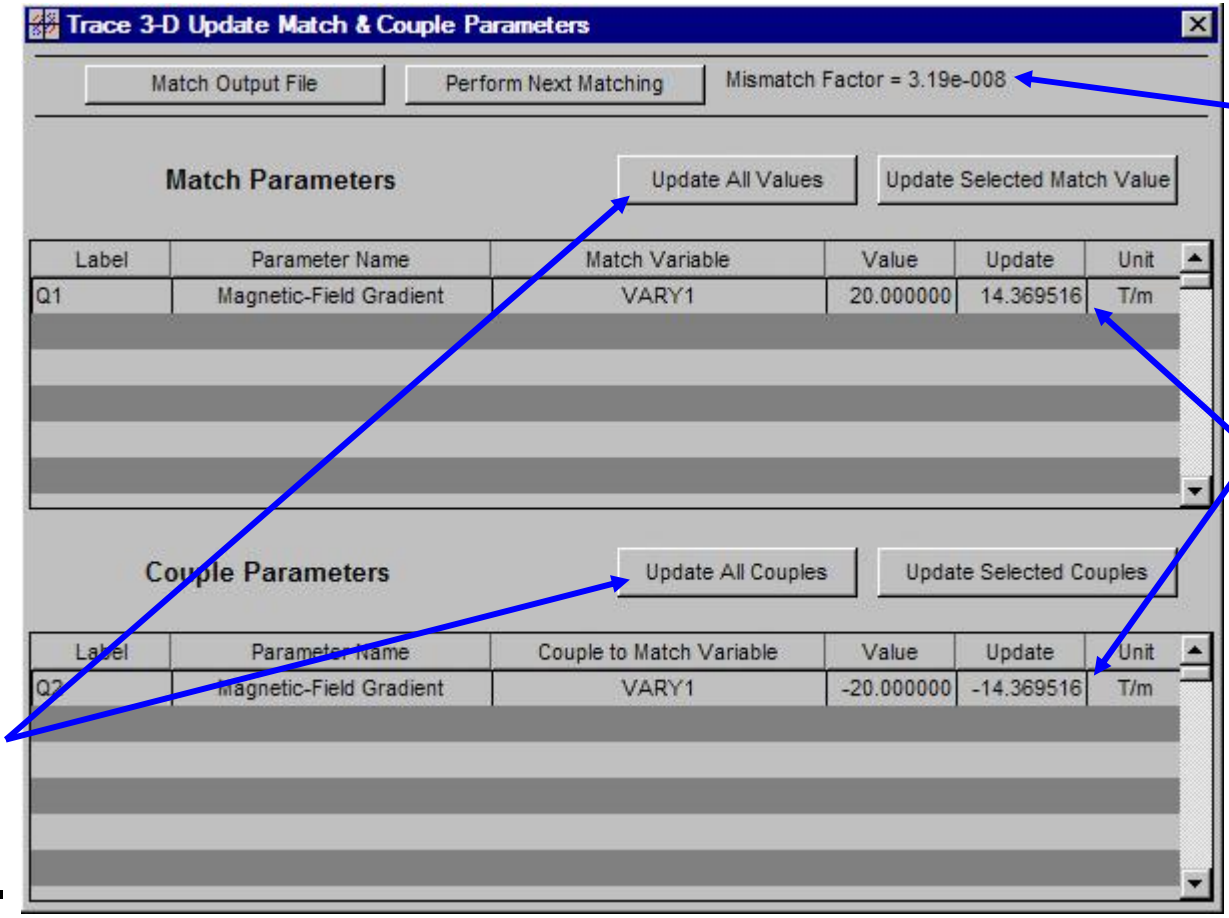
Can You Find A Problem In This Window's Info?

⇒ **Save this setup as " Setup_4_FODO_1"**

Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

4a. Execute a "Perform Matching" TRACE 3-D Command

You should get a point-to-point fit solution that looks like this:



Note MMF

Solution

Update Value & Couple Params.

⇒ **Save this setup as "Setup_4_FODO_3"**

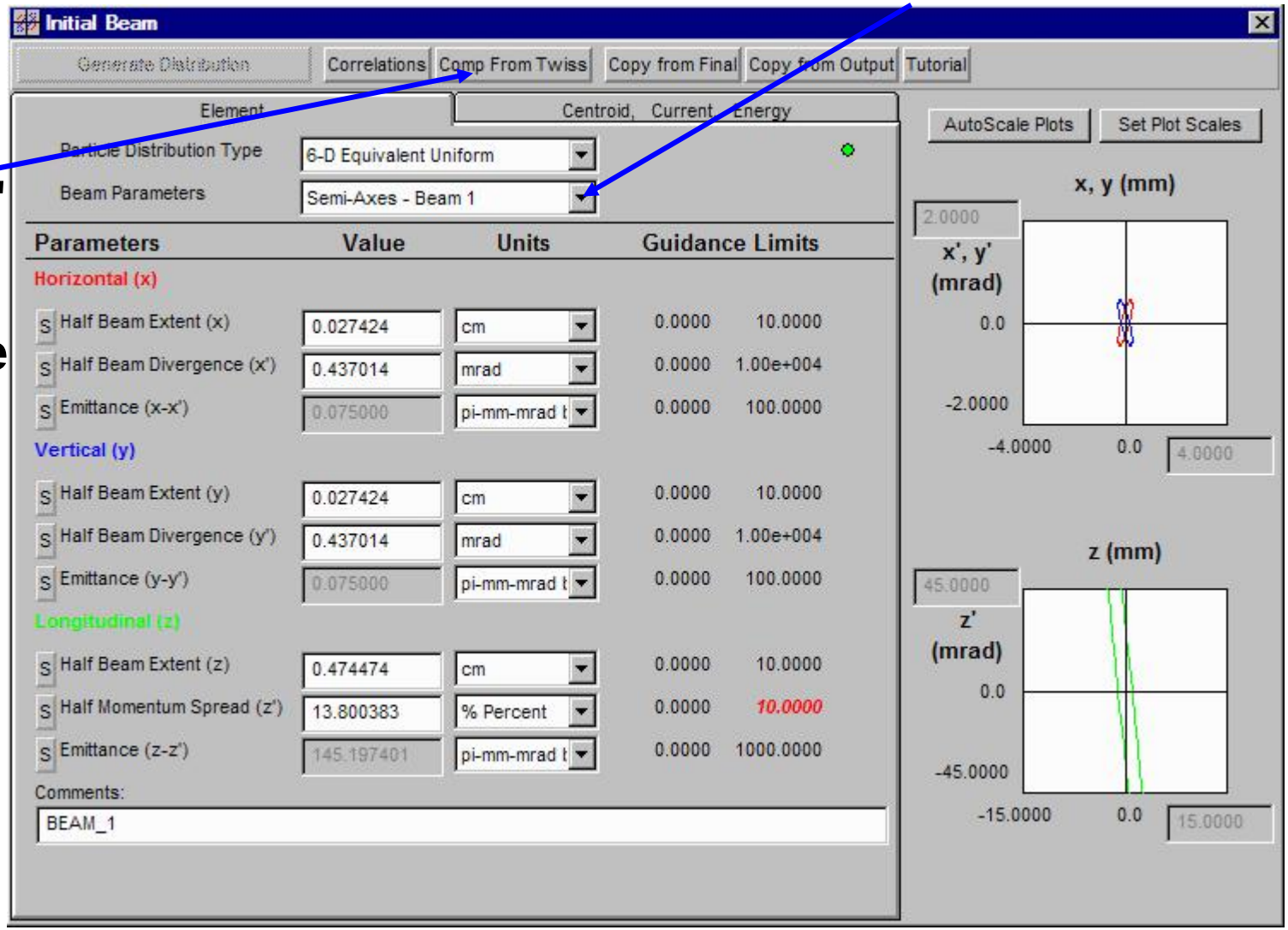
Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

4b. Confirmation of a Point-to-Point Focus in x Direction

Open Beam_1, set Beam Parameters to "Semi-Axes - Beam 1"

Use "Comp From Twiss"

Should Have Equivalent Semi-Axes Data Now



Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

Modify x Semi-Axes Parameters to Approximate a "Point" Beam:

Reduce the Horizontal "Half Beam Extent (x)" by a Factor of 10:

Half Beam Extent (x) = 0.0027424 cm

Increase the Horizontal "Half Beam Divergence (x)" by 10:

Half Beam Divergence (x) = 4.37014 mrad

Execute a "Graph Beam Line" TRACE 3-D Command

Does System Take Approximate "Point" x-Beam to a ~"Point"?

⇒ **Save this example as "Setup_4_FODO_3_Test"**

4c. Make the x-Beam Still More "Point Like"

Change the Horizontal (x) Parameters by Another Factor of 10

Half Beam Extent (x) = 0.00027424 cm

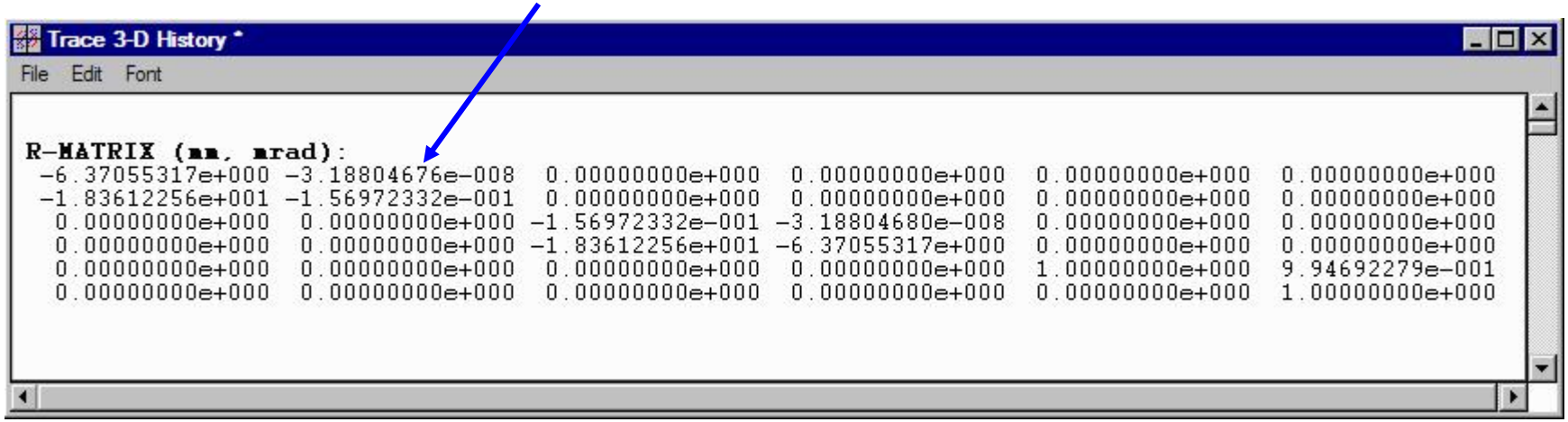
Half Beam Divergence (x) = 43.7014 mrad

Does System Take the More "Point Like" x-Beam to a "Point"?

4d. Does this FODO System Also Have a Matched Beam?

Point to Point, Parallel to Point, Point to Parallel, Parallel to Parallel

- **In example 4 the Quad strengths were solved for a FODO cell (anti-symmetric doublet) to produce a point-to-point focus in x.**
- **Can Directly Verify the Solution Has Condition:** $R_{12} = 0$
  - ⇒ **Use example "Setup_4_FODO_3_Test"**
  - **Execute a "Graph Beam Line" TRACE 3-D Command**
  - **Execute a "Show R Matrix" TRACE 3-D Command**



- **Look at the** R_{34} **matrix element in Trace 3-D History window.**
  - ⇒ **What can you conclude?**

Homework: 4.1 FODO for Parallel-to-Parallel Focus. Name _____

Using TRACE 3-D solve the Setup_1_FODO_1 example for a parallel-to-parallel focus in x.

Start with the PBO Lab File previously saved:
"Setup_3_FODO_1"

Set a TRACE 3-D Match Specification to achieve a parallel-to-parallel focus in x.

⇒ Save this example as "Setup_5_FODO_1"

4.1(a) Use TRACE 3-D Command: Perform Matching. In order to get a good match, do you need to use the Command more than once?

Your Answer: Yes __ No __ If Yes, how many iterations used __

4.1(b) What values did you get for the MMF and the quadrupole gradients?

Your Answer: MMF = _____ , Q1 Grad = _____, Q2 Grad = _____

⇒ Save your solution file as "Setup_5_FODO_4"

4.1(c) What are the results for the 2×2 $R_{xx'}$ submatrix elements?

Your Answer: R_{11} = _____ , R_{12} = _____
R_{21} = _____ , R_{22} = _____

4.1(d) Do these quadrupole gradients also achieve a parallel-to-parallel focus in y?

Your Answer: Yes __ No __

Homework: 4.2 FODO for Point-to-Parallel Focus. Name _____

Using TRACE 3-D solve the Setup_1_FODO_1 example for a point-to-parallel focus in x.

Start with the PBO Lab File previously saved:
"Setup_3_FODO_1"

Set a TRACE 3-D Match Specification to achieve a parallel-to-parallel focus in x.

⇒ Save this example as "Setup_6_FODO_1"

4.2(a) Use TRACE 3-D Command: Perform Matching. In order to get a good match, do you need to use the Command more than once?

Your Answer: Yes __ No __ If Yes, how many iterations used __

4.2(b) What values did you get for the MMF and the quadrupole gradients?

Your Answer: MMF = _____ , Q1 Grad = _____, Q2 Grad = _____

⇒ Save your solution file as "Setup_6_FODO_5"

4.2(c) What are the results for the 2×2 $R_{xx'}$ submatrix elements?

Your Answer: R_{11} = _____ , R_{12} = _____
R_{21} = _____ , R_{22} = _____

4.2(d) Do these quadrupole gradients also achieve a point-to-parallel focus in y?

Your Answer: Yes __ No __